Statistical Aspects of Incremental Sampling

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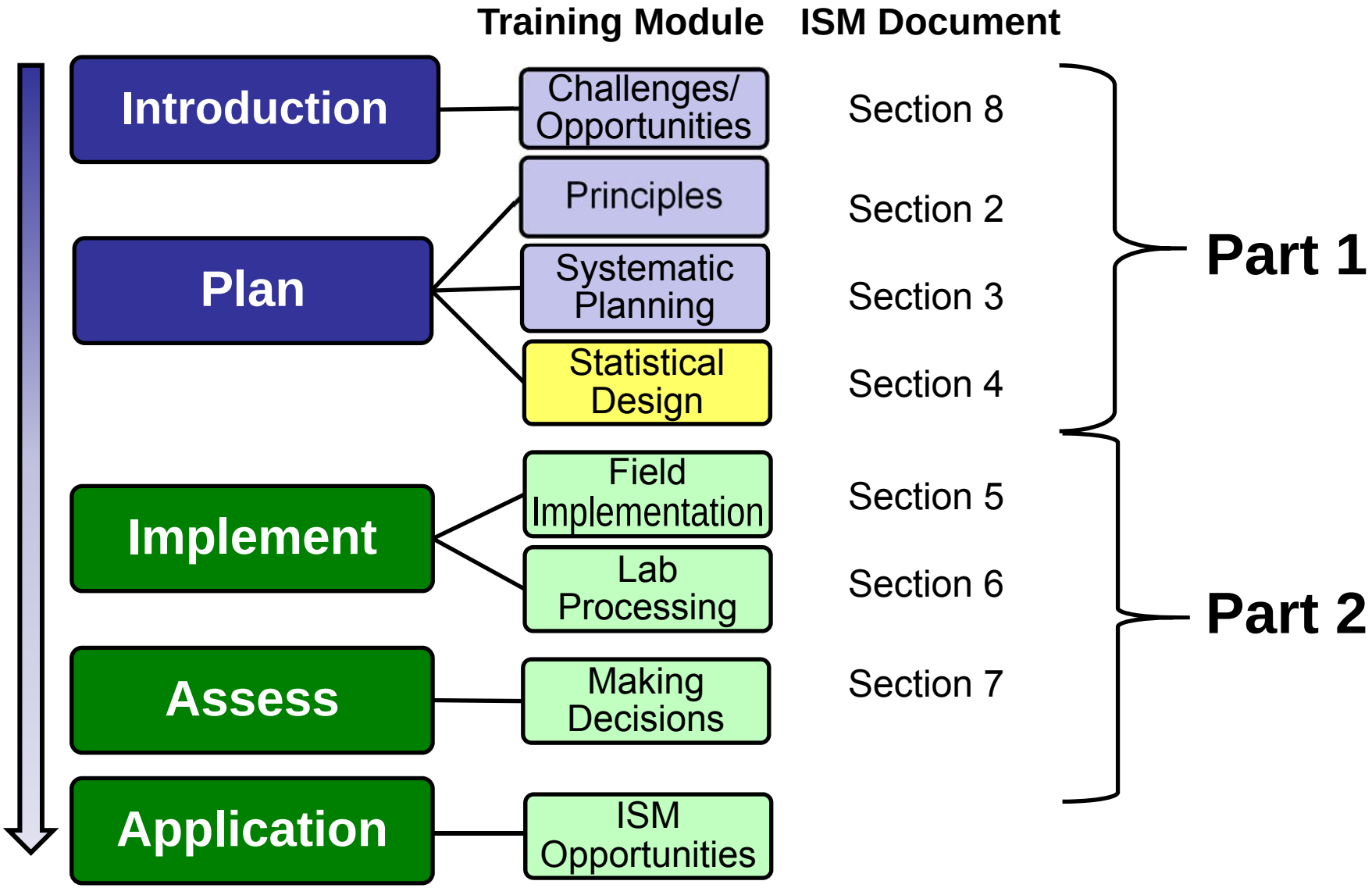
ISM Document and Training Roadmap



Questions – Data Analysis

1. Does a single ISM sample provide a *reasonable* estimate of the mean?

Section 4.2.1

2. Can a 95UCL be calculated with ISM data?

Section 4.2.2

95UCL = 95% Upper Confidence Limit of the mean

Questions – Sampling Design

3. What sampling design should I use?

Section 4.3.4.2

4. Is it reasonable to assume that concentrations are similar across DUs?

Section 4.4.2

Questions – Comparison of Data

5. Can background and site data be compared using ISM?

Sections 4.4.3.3 and 7.2.4

6. Can ISM data be compared to discrete or composite data?

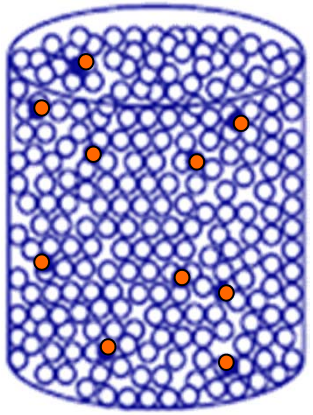
95UCL = 95% Upper Confidence Limit of the mean

1. Does a single ISM provide a reasonable estimate of the mean?

Answer:

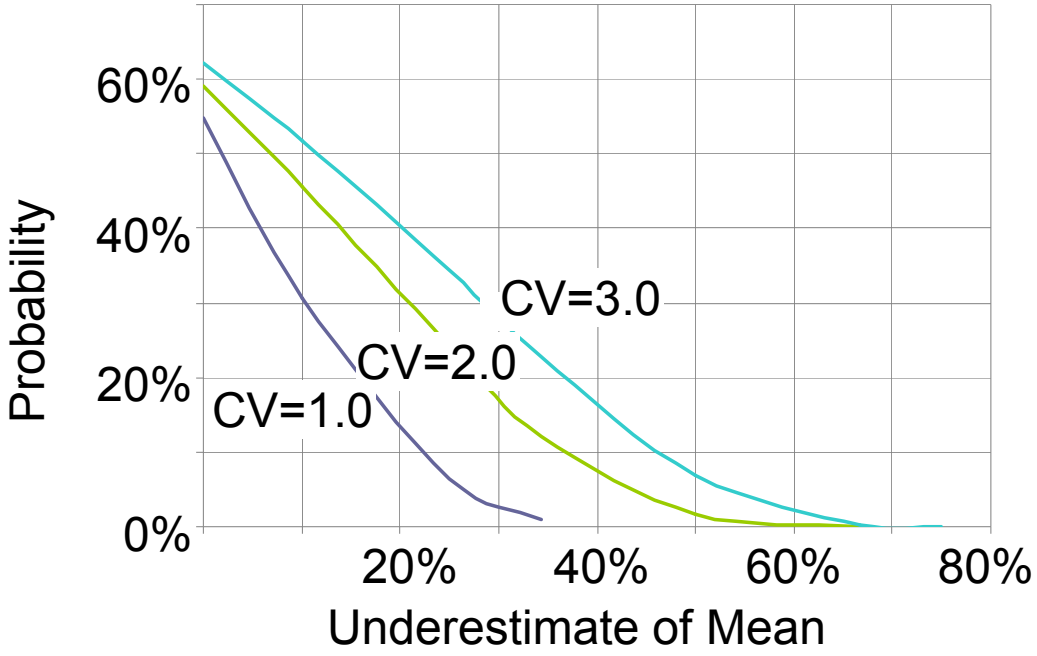
It depends how much error we are willing to accept.

- Why would someone collect just 1 ISM?
  - UCL not required by regulator
  - Save time and expense
  - Assumption that more samples wouldn't change the decision. For example
    - Variance among individual increments is low
    - Mean of DU is far above or below an action level



ITRC, ISM-1, Section 4.2.1

1(b). How "badly" might I underestimate the mean?



CV	Frequency	Magnitude	True Mean	Estimate
1	1 out of 4	15%	400 ppm	≤ 360 ppm
2	1 out of 4	25%	400 ppm	≤ 320 ppm
3	1 out of 4	30 - 60%	400 ppm	160 - 280 ppm

***Coefficient of variation (CV) = St Dev / mean**

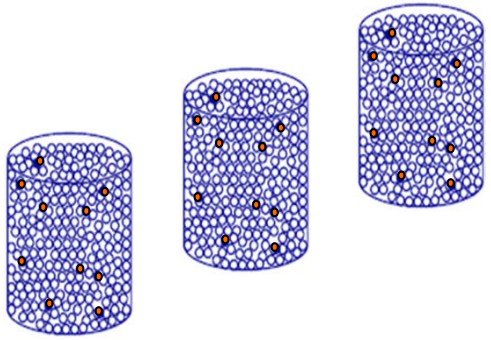
ITRC, ISM-1, Section 4.2.1, Figure 4-2

2. Can a 95UCL be calculated?

Answer:

Yes, even with as few as 3 ISM samples.

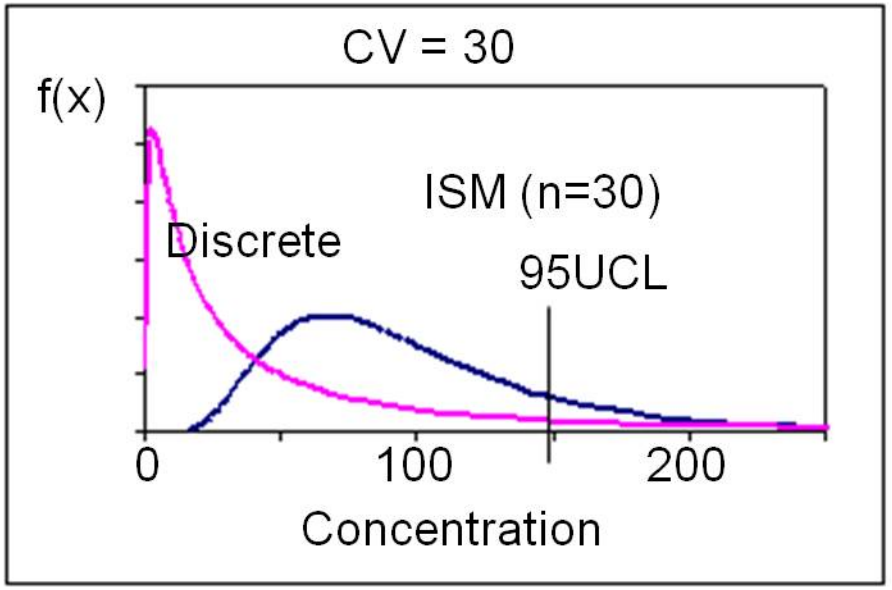
- Need at least 3 replicates ($r \geq 3$)
- Fewer methods are available than we are used to with discrete sampling:
  - Student's-*t*
  - Chebyshev



- Each ISM result provides an estimate of the mean ("x-bar")
- Parameter estimates are calculated directly from ISM data

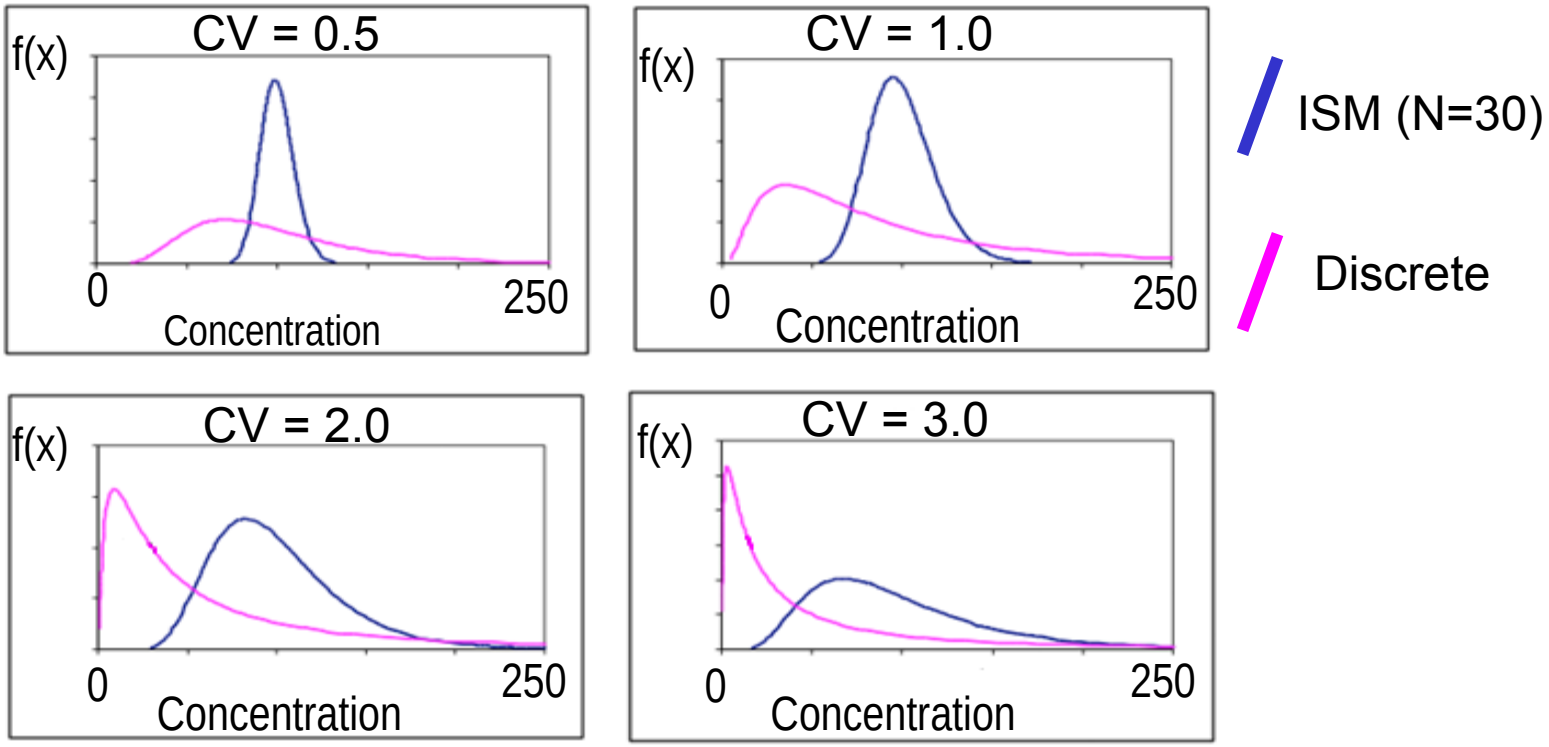
2(b). How do I choose a UCL method?

- Consider performance measures (informed by simulation study)
  - Coverage (probability UCL > mean)
  - Magnitude of difference between UCL and mean



- Recognize the key to performance is variability
  - Distribution of discretes ≠ Distribution of ISM results

Distribution of Means (ISM Replicates)



- ISM distribution variance is smaller
- ISM distribution shape becomes more non-normal with increasing CV of discrete distribution

ITRC, ISM-1, Figure 4-3

Coverage Probabilities

UCL Method	Dispersion Among Individual Increments		
	Low (CV <1.5 or GSD <3)	Medium (1.5 < CV < 3 or 3 < GSD < 4.5)	High (CV >3 or GSD >4.5)
Student's-*t*	Yes	No	No
Chebyshev	Yes	Yes	Maybe

- Both methods provide desired 95% coverage when variability is low
- Chebyshev has more consistent 95% coverage for medium and high variability
- Increasing *r* (>3) and n (>30) provides marginal improvement

ITRC, ISM-1, Table 4-4, Sections 4.3; Appendix A

How much does Chebyshev overestimate?

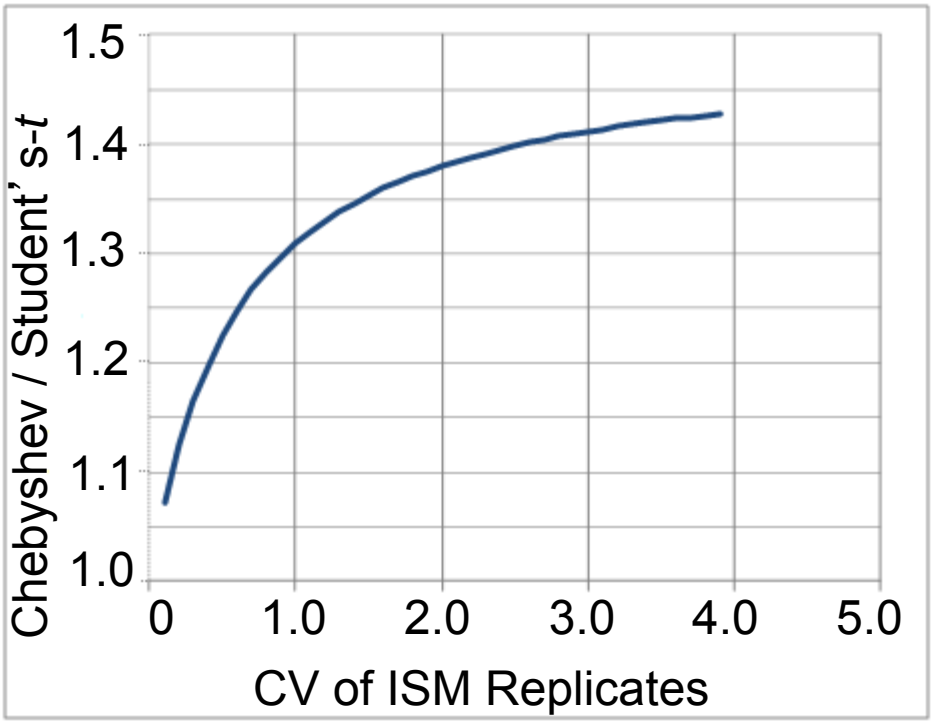
- Chebyshev will tend to yield 10-45% higher UCLs than Student's-*t* depending on the CV of the replicates
- Example: Student's-*t* = 100 ppm, Chebyshev = 110 -145 ppm

Student's-*t*

$$UCL = \overline{X} + t_{1-\alpha,r-1} \times \frac{s_{\overline{x}}}{\sqrt{r}}$$

Chebyshev

$$UCL = \overline{X} + \left(\sqrt{\frac{1}{\alpha} - 1}\right) \frac{s_{\overline{x}}}{\sqrt{r}}$$



ITRC, ISM-1, Section 4.3.1.1

2(c). Can I use ProUCL to calculate the 95UCL?

Answer:

No. However, there are other tools available to calculate a 95UCL from ISM data.

- ProUCL is designed to work with discrete sample data.
- ISM replicates are fundamentally different from discrete samples, and data are typically available for only a few replicates.
- The ITRC guidance document has a link to a calculator that works for ISM data.

ITRC, ISM-1, Sections 4.2.2

2(d). What does the variability in ISM reveal?

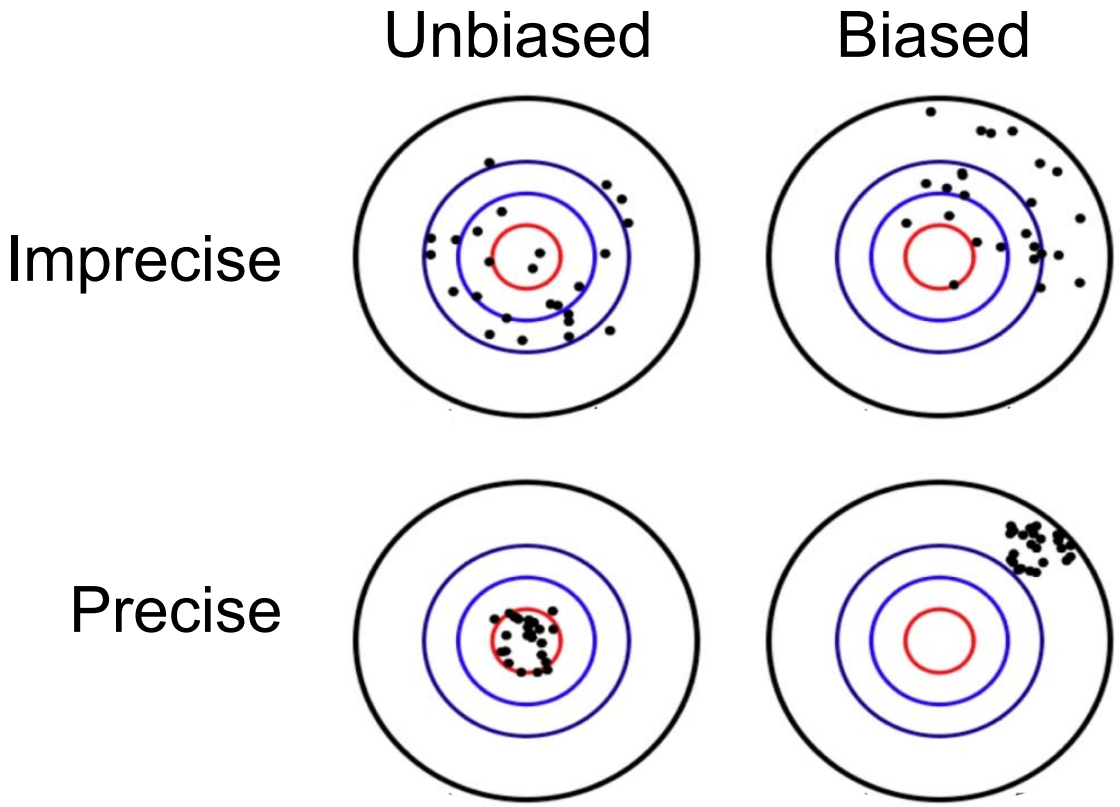
Answer:

High variability can indicate lab error. Low variability does not ensure that the results are sufficiently accurate to avoid decision error.

- RSD is the ratio of statistics calculated from ISM replicates
  - RSD = SD / mean
- If the goal is to make sure that the mean is not underestimated, a 95UCL should be calculated regardless of whether the RSD is high or low

ITRC, ISM-1, Section 4.3.4.4

2(d). What does the variability in ISM reveal?



- Accuracy reflects both bias and precision (reproducibility)

ITRC, ISM-1, Section 4.3.1, Figure 4-6 and Appendix E

3. Is there a preferred ISM sampling design?

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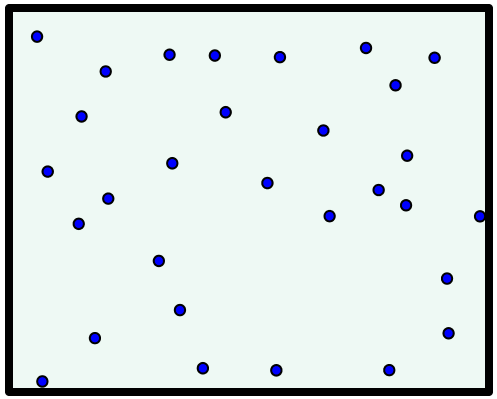
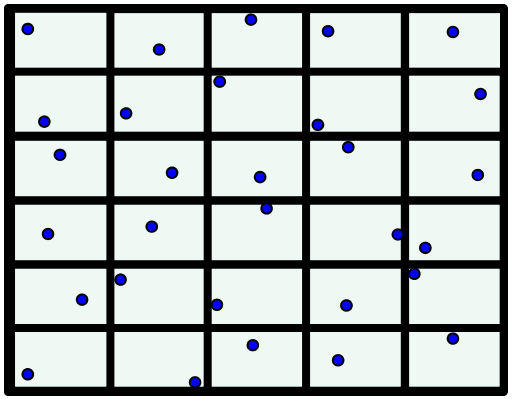
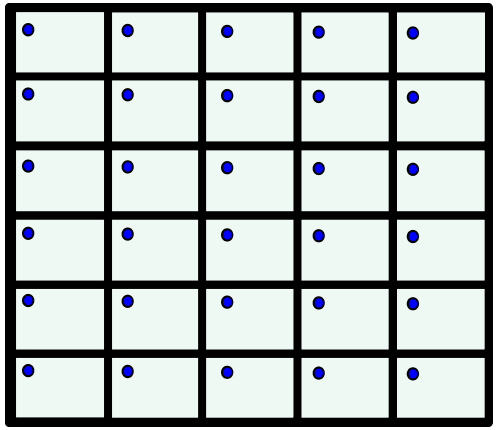


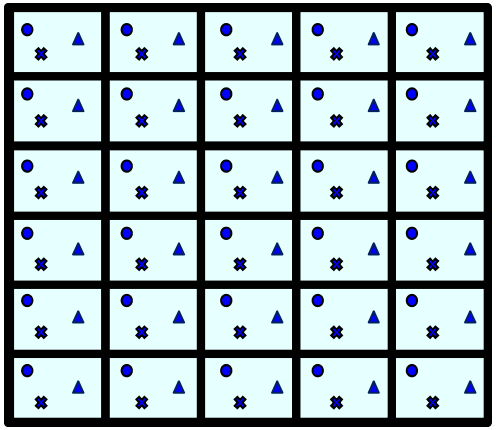
Simple Random



Random within Grid



Systematic



Systematic (3 replicates)

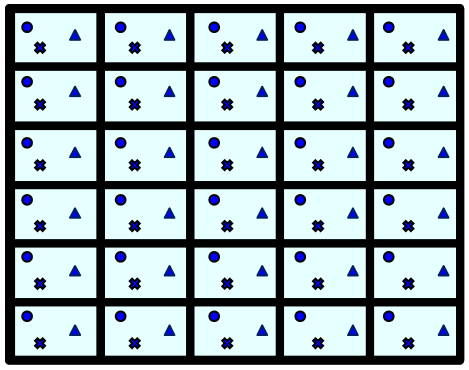
ITRC, ISM-1, Section 4.3.4.2

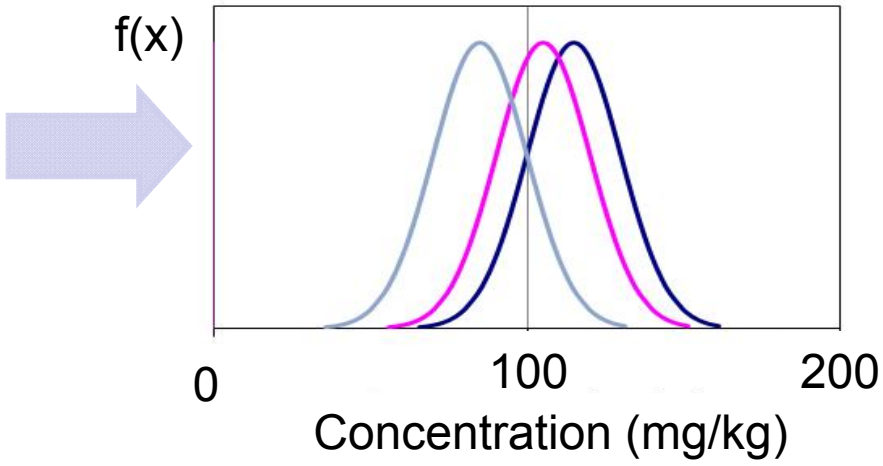
3. Is there a preferred ISM sampling design (continued)?

Answer:

Each random sampling design yields unbiased estimates of the mean and is an acceptable approach in most situations.

- Systematic random sampling is most often used because it is the easiest to implement





ITRC, ISM-1, Section 4.3.4.2

3(b). How many increments?

Answer: n = 30

- As the number of increments increases:
  - spatial coverage improves
  - variability between ISM results decreases
  - 95UCL will tend to be closer to the mean
- Size of DU can be a consideration – large DUs may require more increments

10 20 30 40 50 60 70 80 90 100

ITRC, ISM-1, Section 4.3.4.1

3(c). How many replicates?

Answer: r =3

- For most DUs, 3 is sufficient.
- Minimum number to calculate standard deviation (and 95UCL) of ISM results

- More replicates will produce a 95UCL closer to the actual mean, but may not be cost-effective unless the result is near the action level

ITRC, ISM-1, Section 4.3.4.1

4. Can I extrapolate results across DUs?

- Unsampled DU – extrapolate estimate of mean
- DU with 1 ISM – extrapolate estimate of variability
  - Standard deviation (SD)
  - Coefficient of variation (CV)

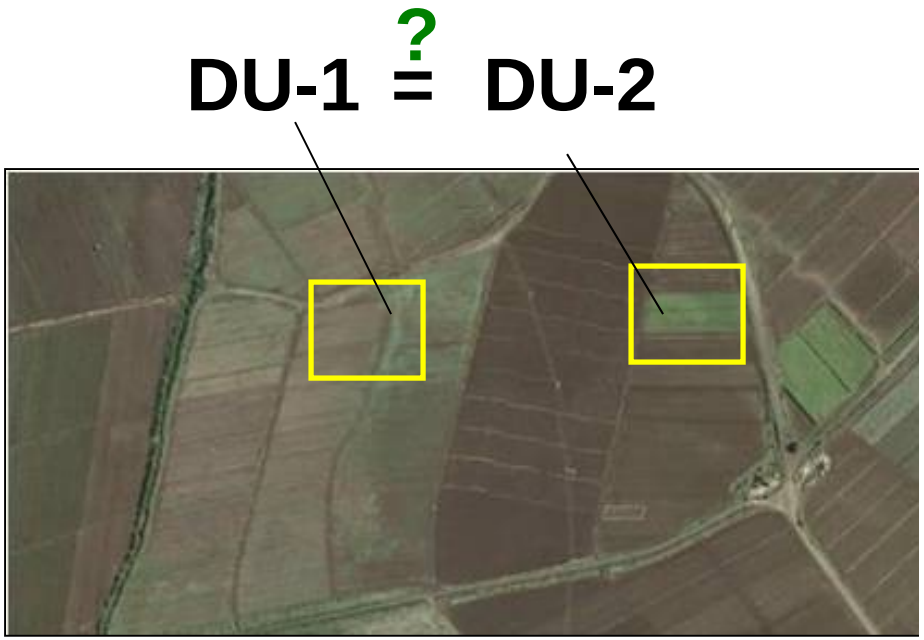
DU-1 ? DU-2

ITRC, ISM-1, Section 4.4.2

4(b). Extrapolation of the Mean

Answer:

Assumes the mean concentration in the unsampled DU(s) is the same as the sampled DU.



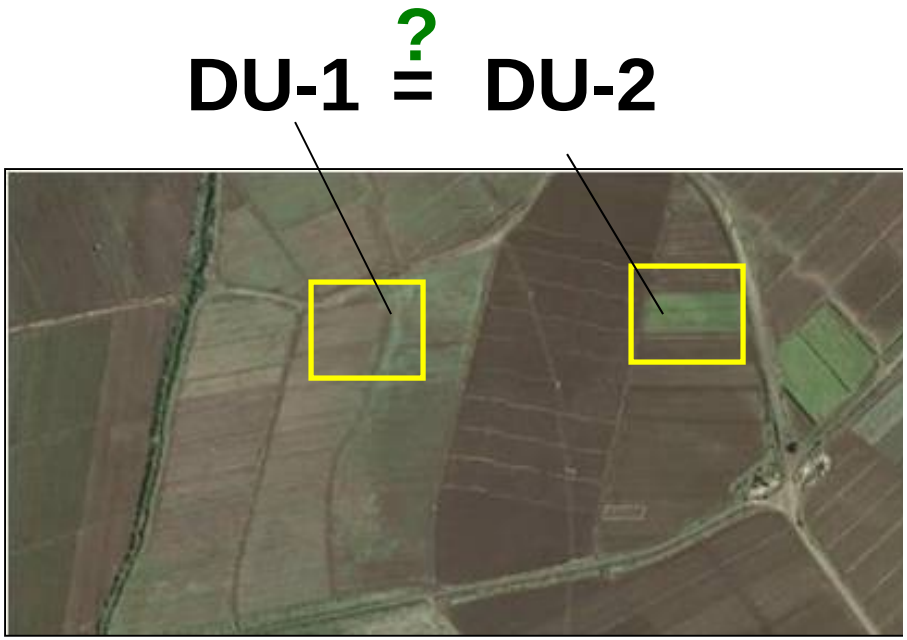
- DU-1:
  - Mean = 100
  - SD = ?
- DU-2:
  - Assumed Mean = 100

ITRC, ISM-1, Section 4.4.2

4(c). Extrapolation of the Variance

Answer:

Assumes that the variability is similar in all DUs.



- DU-1:
  - Mean = 100
  - SD = 50
  - CV = 50/100 = 0.5
- DU-2:
  - Mean = 400
  - Assumed CV = 0.5 = SD / 400, therefore SD = 200

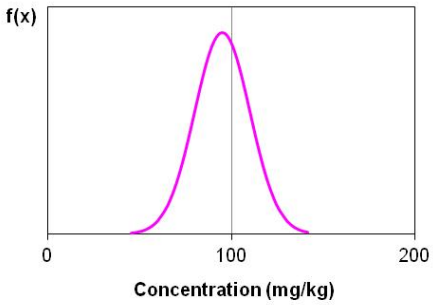
ITRC, ISM-1, Section 4.4.2

5. Can background and site ISM data be compared?

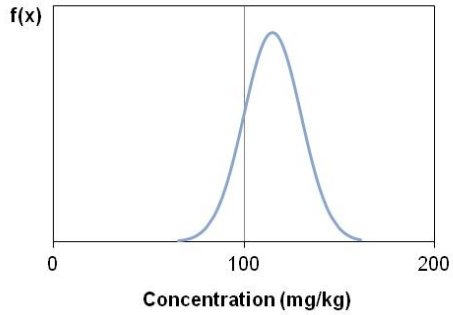
Answer:

Yes, but statistical tools for comparison are limited.

Background



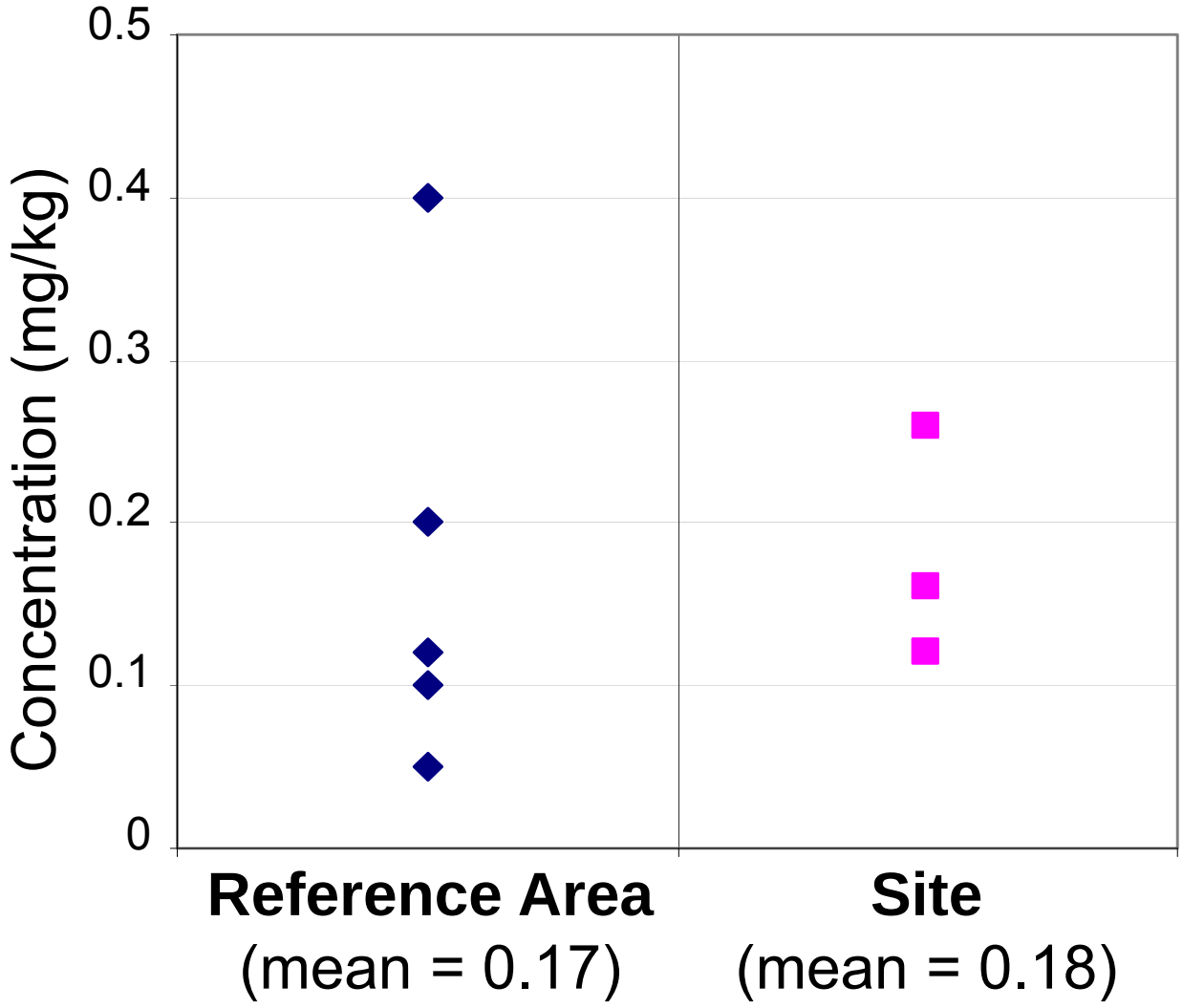
DU-1



- Is the site mean equal to background?
- Hypothesis testing is limited
- Cannot test upper tails with ISM data

ITRC, ISM-1, Section 4.4.3.3

5. Example Background Comparison



ITRC, ISM-1, Section 7.2.4, Figure 7-1

6. Can ISM and discrete or composite data be compared?

Answer:

Yes, if the sample support is known.

- Measurement variability is a function of sample support
- Comparisons are possible with caution and consideration of all differences in sampling and analysis methods and meaning of data
  - Sample collection
  - Sample prep
  - Analysis method
  - ISM estimates mean

ITRC, ISM-1, Section 4.4.3.3

Summary: Statistical Design

- Mean or 95UCL from ISM data may be used to make decisions
- 3 replicate samples provide adequate information for a 95UCL
- Systematic random sampling is most commonly used
- About 30 increments per ISM sample is usually sufficient
- Extrapolation of the mean or variance can be very uncertain
- Comparisons between ISM site and background data are possible, with caution
- Comparisons between ISM and other data are possible, with caution

ISM Part 1 Summary and Part 2 Preview